

UNIFORM CIRCULAR MOTION

IB-PHYSICS SL/HL

STUDENT'S NAME :

Nature of Science : The ideas about circular motion came from observations from universe. Question came up was what kept the astronomical objects in place in circular motion. Scientists were able to deduce that there must be a force acting radially inward for circular motion. Same concepts of physics are applied whether it is bicycle moving in circular path or planets orbiting.

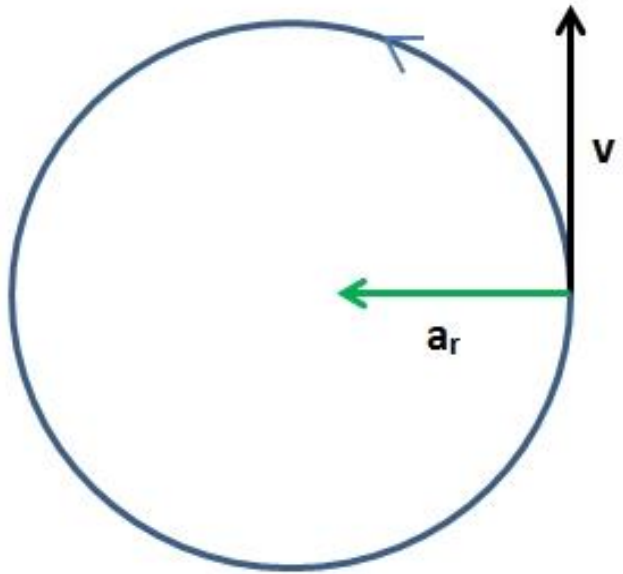
Understanding :

Frequency, Period , angular displacement , angular velocity, centripetal acceleration, centripetal force

Applications and Skills :

- Identifying the forces providing the centripetal forces such as tension, friction, gravitational, magnetic
- Solving problems involving centripetal force, centripetal acceleration, period, frequency, angular displacement, angular speed.
- Describing examples of horizontal and vertical circular motion like looping the loop, the wall of death

UNIFORM CIRCULAR MOTION

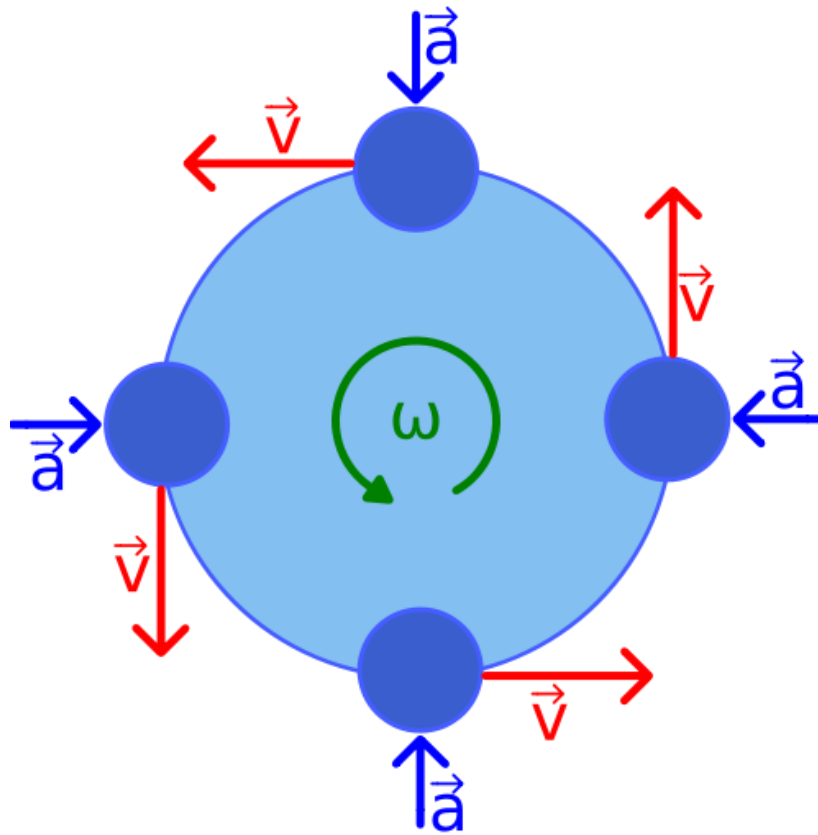


- It is defined as the motion of the particle along the circumference of circle with constant speed

OR

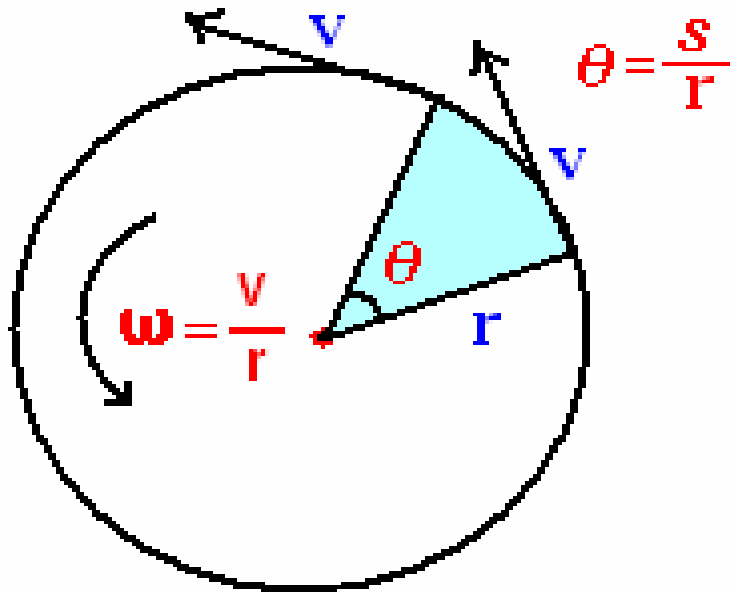
- It is defined as the periodic motion of the particle along the circumference with constant angular speed.

IN UCM, THE ACCELERATION IS ALWAYS DIRECTED TOWARDS THE CENTRE.



- As shown in fig. in uniform circular motion the magnitude of velocity is constant but its direction is constantly changing along its path.
- This change in direction of velocity produces acceleration which is always directed towards the centre.

ANGULAR DISPLACEMENT & ANGULAR VELOCITY

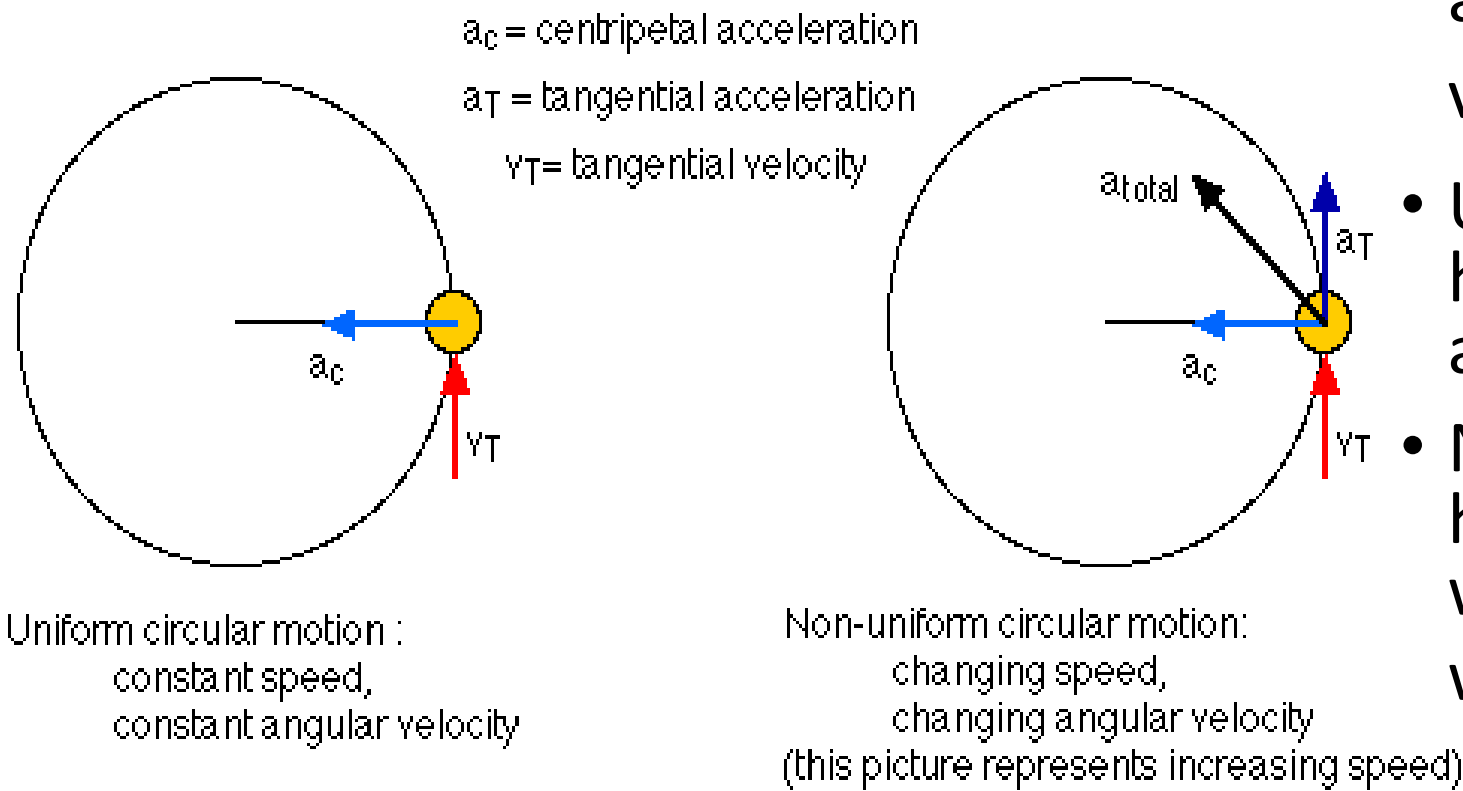


- **Angular displacement θ** is defined as an angle through which an object moves in a circular path.
 - It is the angle in radian between the initial and final position
- $\theta_f - \theta_i = \theta = \frac{s}{r}$, r is the radius of the circle, s is the length of the arc subtended by angle θ
- **Angular velocity ω** is defined as the rate of change of angular displacement.

$$\omega = \frac{d\theta}{dt} = \frac{d\left(\frac{s}{r}\right)}{dt} = \frac{ds}{dt \cdot r} = \frac{v}{r}, \quad v \text{ is linear velocity.}$$

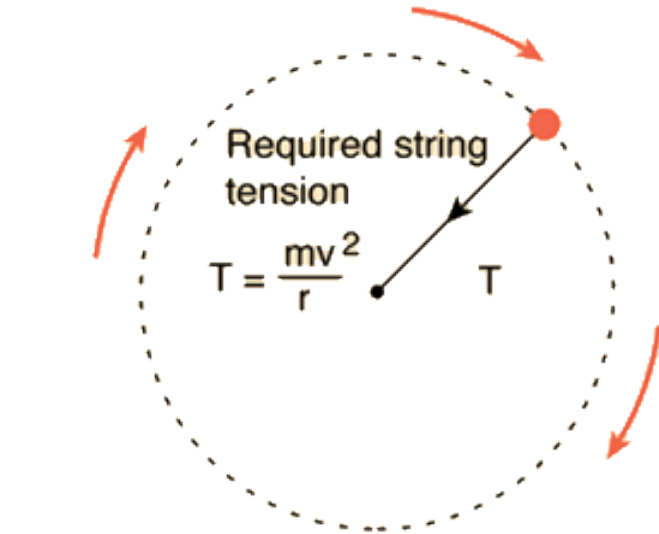
Thus greater the distance of the particle from the centre, greater is the linear velocity.

CENTRIPETAL/ANGULAR ACCELAERATION

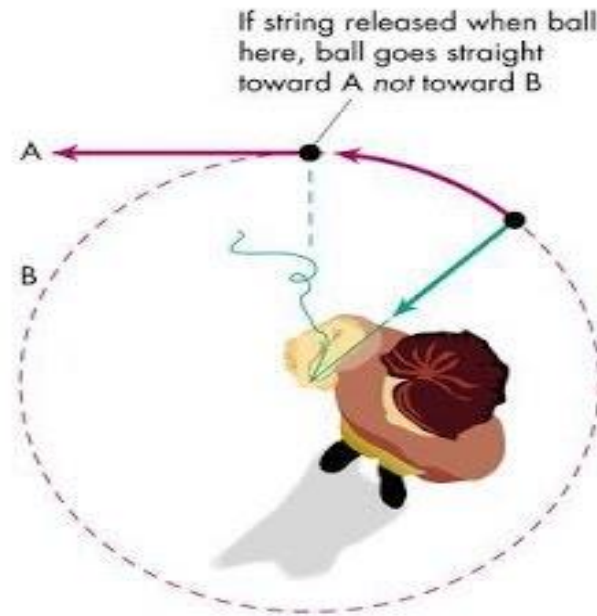


- **Angular acceleration** is defined as the rate of change of angular velocity $a_c = \frac{d\omega}{dt} = \frac{v^2}{r} = r\omega^2$
- Uniform circular motion will have only centripetal acceleration WHILE
- Non-uniform circular motion will have tangential acceleration as well. The total acceleration here will be $a = \sqrt{a_c^2 + a_T^2}$

CENTRIPETAL FORCE



Side view



Top view

- A body performing circular motion is always acted upon by a force which is directed towards its centre. This force is called centripetal force.

- Examples:

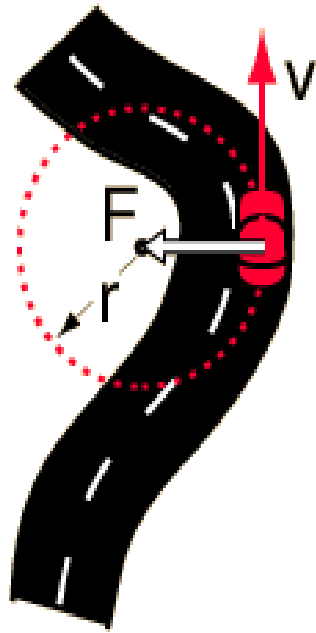
1. when a stone is tied with a string and moved in circular motion, the necessary centripetal force is supplied by tension in the string. And its value is :

$$\Rightarrow F = ma = m \frac{v^2}{r}$$

Centripetal force -continue

$$F_{\text{centripetal}} = m \frac{v^2}{r}$$

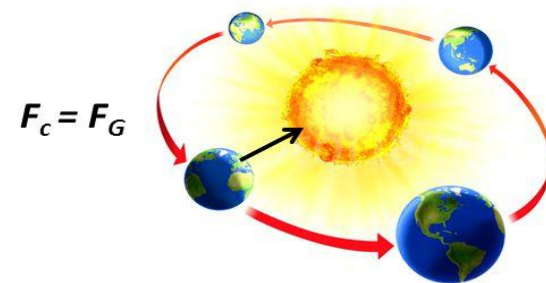
$\frac{v^2}{r}$ is the centripetal acceleration



2. In case of a car taking a turn, the necessary centripetal force is supplied by friction between the road and tyres.

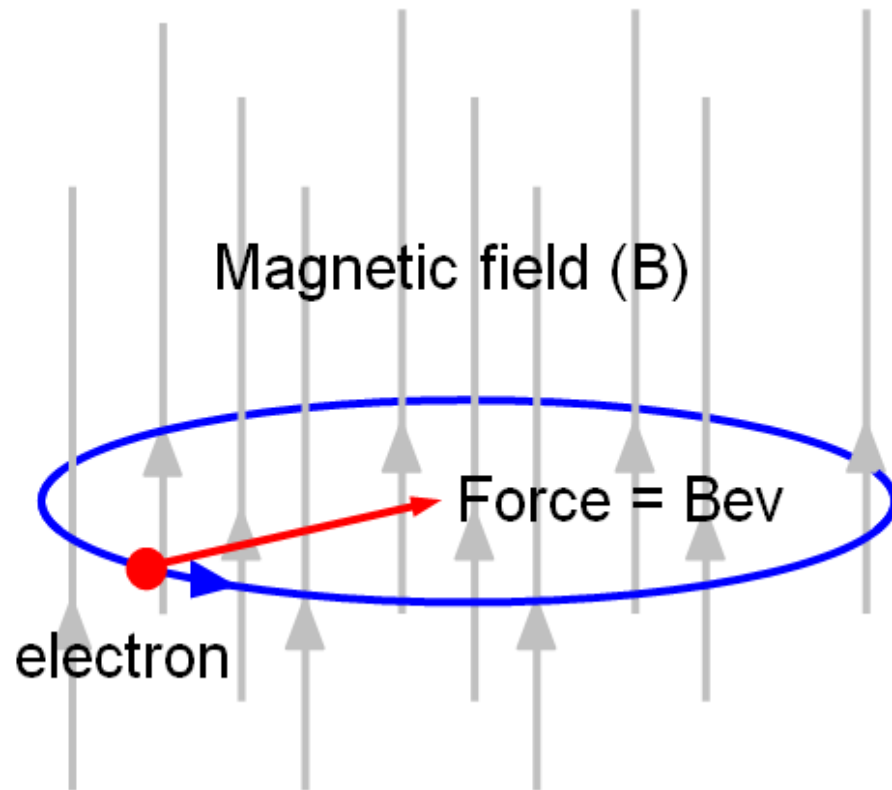
2. In case of satellites revolving around the earth, the necessary centripetal force is provided by gravitational pull of the earth. Centripetal Force and Gravity

- For an object (like a satellite) in circular motion due to gravity

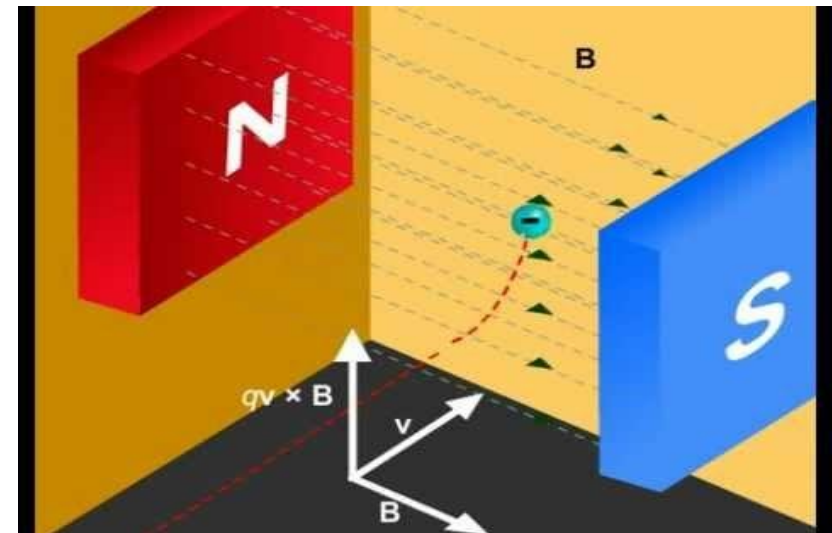


$$F_c = F_G$$
$$ma_c = G \frac{Mm}{r^2}$$
$$v^2 = \frac{GM}{r}$$

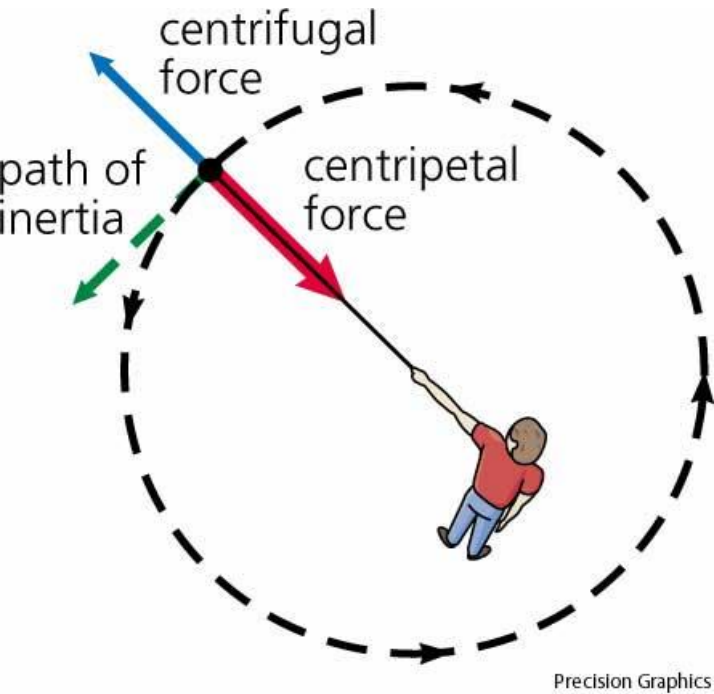
Centripetal force -continue



- An electron entering a perpendicular magnetic field will follow a circular motion. The necessary centripetal force here is due to Lorentz force.



CENTRIFUGAL FORCE



- **Centrifugal force** is pseudo force(false force) in UCM, that is directed along the radius and away from the centre.
- Examples:
 - Driers in washing machine consists of a cylindrical vessel with perforated walls. As the cylindrical vessel is rotated fast, the centrifugal forces water out of perforations and thereby dries wet clothes.
 - The children sitting in merry go round experiences an outward pull as merry go round rotates about vertical axis. This is due to centrifugal force acting on the children.

BANKING OF ROADS

- When a car is moving along a curved road, it is performing a circular motion, since curved road is arc of a circle. Here the necessary centripetal force is provided by the friction between the road and tyre. ie

Centripetal force = frictional force

$$m \frac{v^2}{r} = \mu mg$$

Or $v = \sqrt{\mu rg}$, μ is the coefficient of friction between road and tyres.

And v is the maximum safety with which a car can be safely driven along a curved horizontal road. So if the speed exceeds this limit the car will be thrown off.

In case of cars moving at higher speeds, the force of friction has to be made higher by making the surface rough, but this increases the wear and tear of tyres. Besides force of friction is not always as its value decreases when roads become wet or oily.

- Banking of roads is the best remedy for vehicles travelling at high speed along curved road.

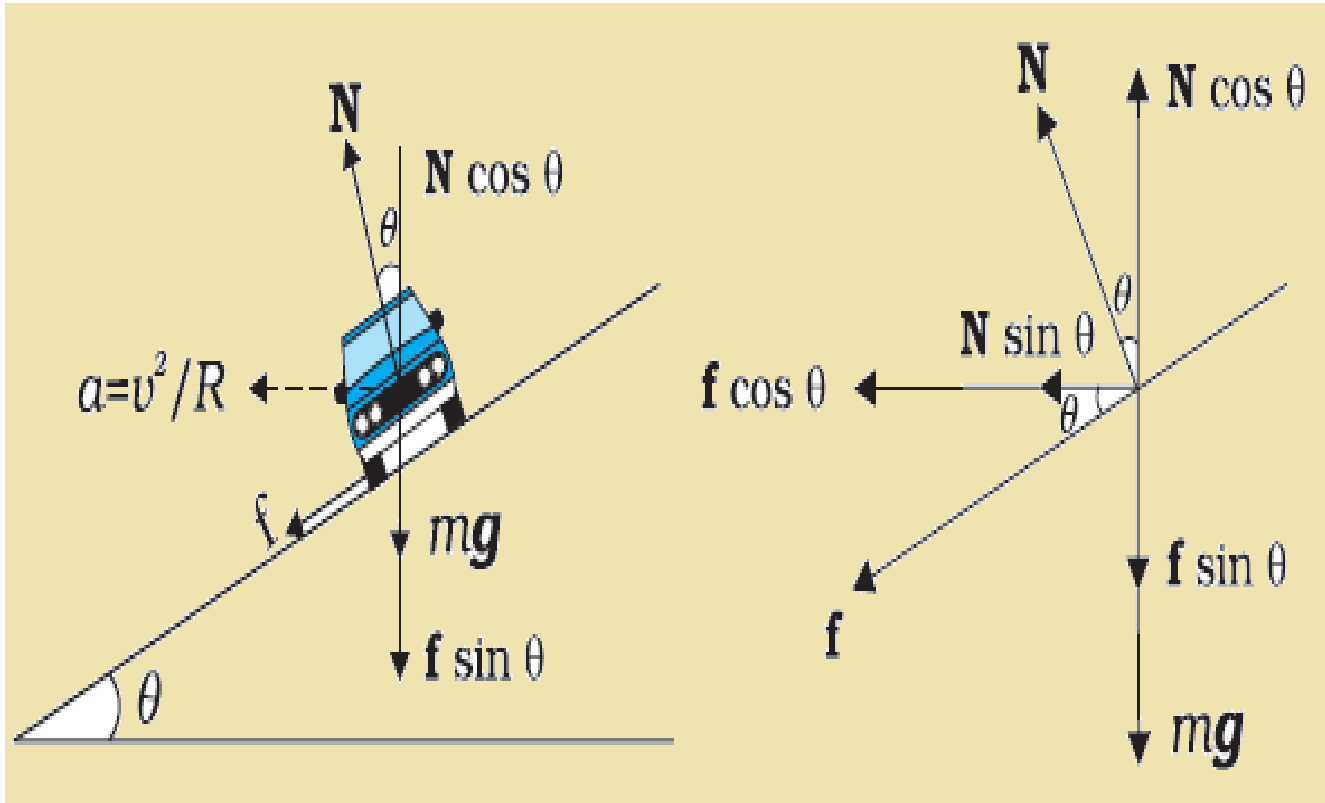
Banked Roads

What is banking of roads?

The process of **raising** the **outer edge** of a road over the inner edge through a certain angle is known as banking of road.



FORCE DIAGRAM OF VEHICLE ON BANKED ROAD



- As shown in fig. the necessary centripetal force is provided by two forces. So the equating two gives :

$$N \sin \theta + f \cos \theta = m \frac{v^2}{r}, \text{ and}$$

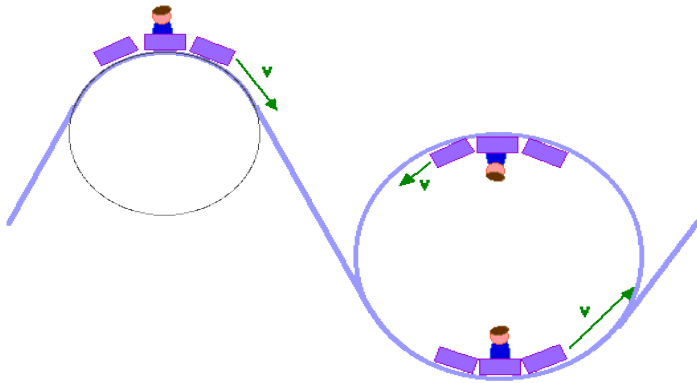
equating vertical forces gives :

$$N \cos \theta = f \sin \theta + mg$$

Solving two equations and simplifying gives :

$$v_{max} = \sqrt{rg \left[\frac{\mu + \tan \theta}{1 - \mu \tan \theta} \right]}$$

VERTICAL CIRCULAR MOTION

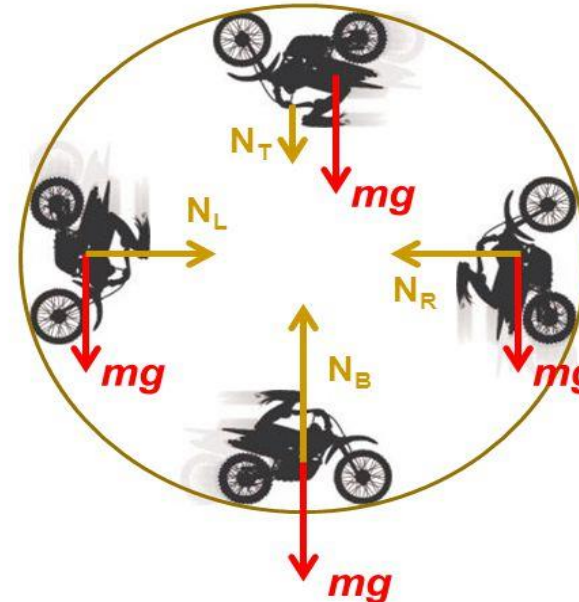


A common misconception here is that the centripetal force is another force like weight, tension, friction etc. however it isn't another force it is the name of the resultant unbalanced force towards the centre.



Vertical Circular Motion

Gratuitous Hart Attack



$$\sum F = \frac{mv^2}{r}$$

$$N_B - mg = \frac{mv^2}{r}$$

$$N_L = N_R = \frac{mv^2}{r}$$

$$N_T + mg = \frac{mv^2}{r}$$

What minimum speed must she have to not fall off at the top?

LOOPING THE LOOP AND ENERGY CONSERVATION.

Simple physics

From **A** to **B**, conserve KE+PE

$$mgh = \frac{1}{2}mV_1^2$$

so $h = V_1^2/2g = 5R/2$

At **C** (top of the loop) centrifugal force exactly balances weight, so

$$mg = mV_2^2 / R$$

or

$$V_2 = \sqrt{gR}$$

From **B** to **C**, conserve KE+PE

$$\frac{1}{2}mV_1^2 = \frac{1}{2}mV_2^2 + 2mgR$$

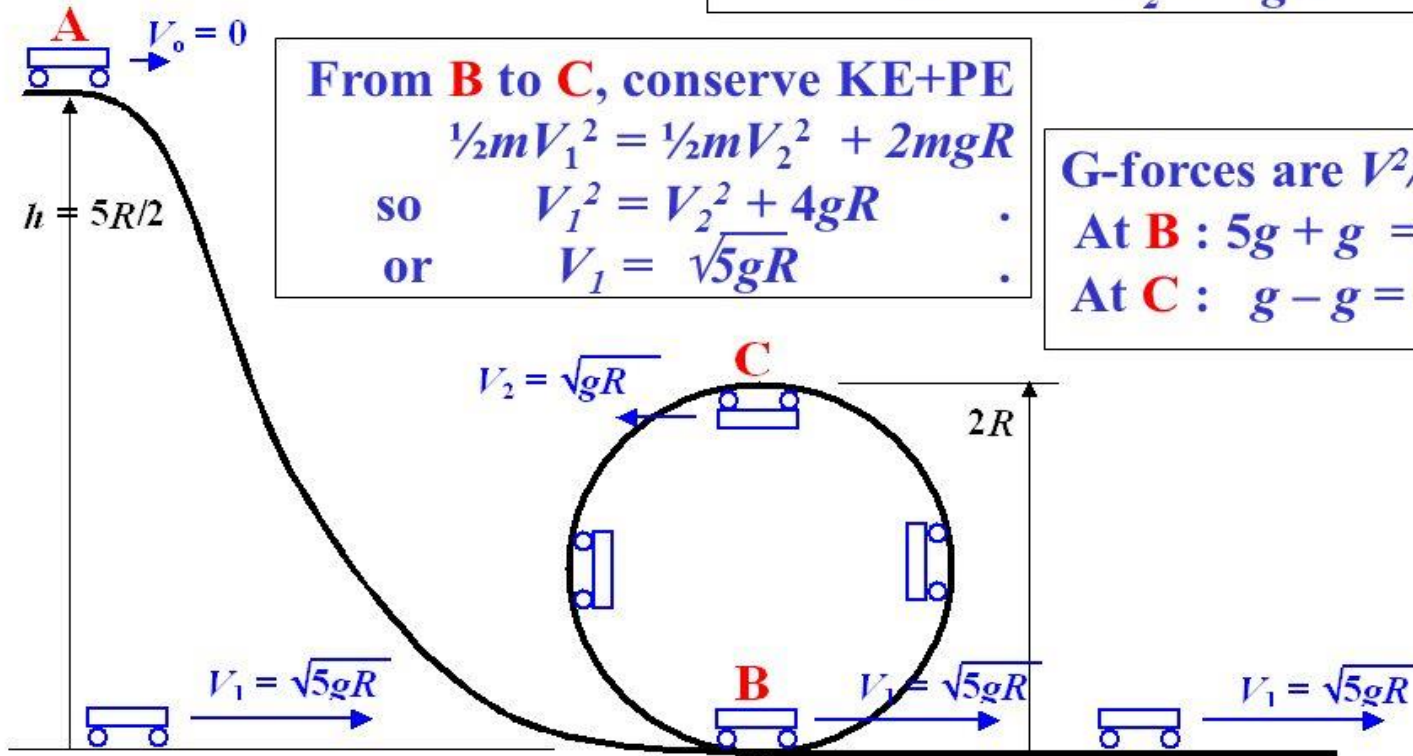
so $V_1^2 = V_2^2 + 4gR$.

or $V_1 = \sqrt{5gR}$.

G-forces are V^2/R :

At **B** : $5g + g = 6g$

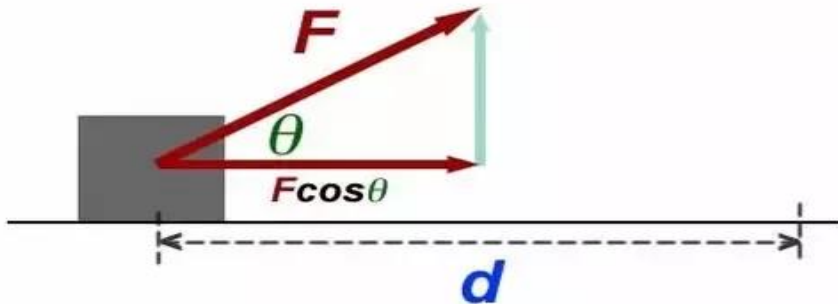
At **C** : $g - g = 0$



WORK DONE IN UCM IS ZERO.

- Work done on an object is the product of force applied and the displacement covered in the direction of force applied. If the force is inclined at an angle then work will be the product of the component of force in the direction of displacement and the displacement.
- In circular motions, or more precisely uniform circular motion, resultant force which is known as centripetal force is directed towards centre while the object moves at right angle to it at any instant. Mathematically, θ will become 90 degrees and $\cos(\theta)$ is zero. Hence, the work done will be zero. Physically, there is no component of force in the direction of motion. Only one component remains which does not do work at all. Hence, the work done in a uniform circular will be always zero.

$$W = Fd \cos \theta$$



PRACTISE QUESTIONS.

An object of mass 450 g moves in a circular path of radius 1.5 m.
It completes 2.5 revolutions per second.

Calculate

- a) the angular velocity
- b) the linear speed of the body
- c) the magnitude of the centripetal force needed to maintain this motion
- d) the work done by this force during 10 revolutions.

A mass of 15 kg is moving in a circular path at the end of a metal rod 4 m long.
The axis of rotation passes through the *other end* of the rod and the plane of the motion is *horizontal*.

The maximum tension that the rod can tolerate is $5 \times 10^4 \text{ N}$.

- a) Draw a diagram showing the force(s) acting on the mass.
Ignore the force of gravity and assume there are no frictional forces.
- b) Indicate clearly on the diagram the direction in which the mass will move if the rod breaks.
- c) Calculate the maximum linear speed with which the mass can move without breaking the rod.
- d) Calculate the maximum rotational frequency corresponding to this maximum speed.

PRACTISE QUESTIONS CONTINUE...

A 200g mass is moving in a circular path on the end of a light metal rod 0.5m long.

The axis of rotation passes through the *other end* of the rod and the plane of the motion is *vertical*.

The rotational frequency is 1.2 s^{-1} .

- Calculate the angular velocity.
- Draw two diagrams showing the positions of the mass when the tension in the rod is i) maximum and ii) minimum.
Label the forces acting on the mass and explain briefly why the tension varies.
- Calculate the magnitudes of the maximum and minimum tensions.
- Calculate the angular velocity at which the minimum tension is zero.

a) At what angle should a road surface be banked in order that a vehicle can go round a bend of radius 55m at a speed of 40 kmh^{-1} ?

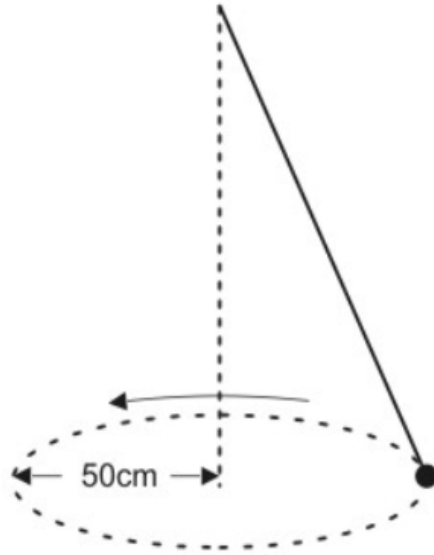
b) Suppose that a vehicle attempts to go round this bend at 100 kmh^{-1} .

If the coefficient of friction between the wheels of the vehicle and the road surface is 0.25, and the mass of the vehicle is 1.5tons (1500kg), will the vehicle skid or not?

Show your calculations.

PRACTISE QUESTIONS CONTINUE...

A body of mass 2 kg is attached to a string 1 m long and moves in a horizontal circle of radius 50 cm, as shown below.



This arrangement is often called a "conical pendulum".

- a) Calculate the magnitude of the tension in the string.
- b) Calculate the time period of the motion (the time to complete one revolution).

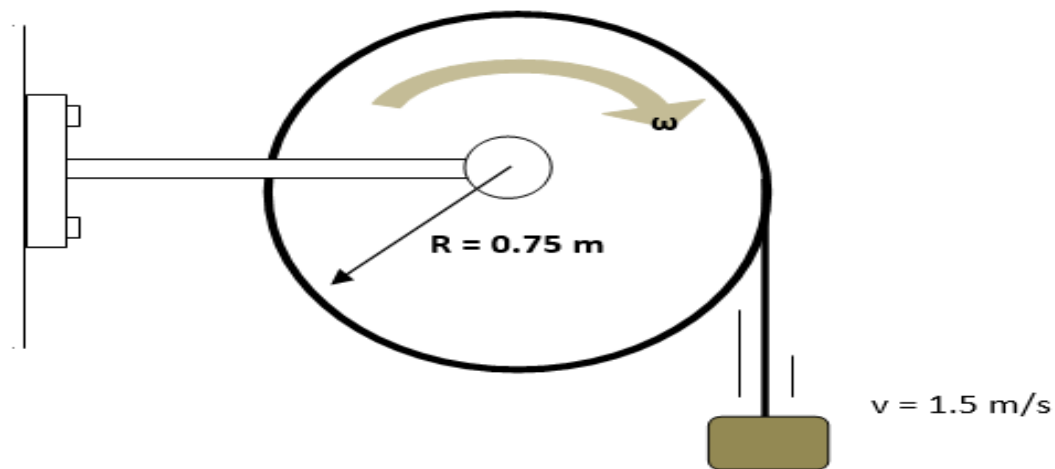
A body, of mass m , is moving in a circular path of radius r
The centripetal force acting on it is F_c

Show that the rotational frequency of the motion is given by

$$f = \frac{1}{2\pi} \sqrt{\frac{F_c}{mr}}$$

PRACTISE QUESTIONS CONTINUE...

A horizontal cylinder with a radius of 0.75 m has a rope wrapped around it that supports a weight at its end as shown. At what constant angular speed must the shaft rotate if the weight is to descend at 1.5 m/s?



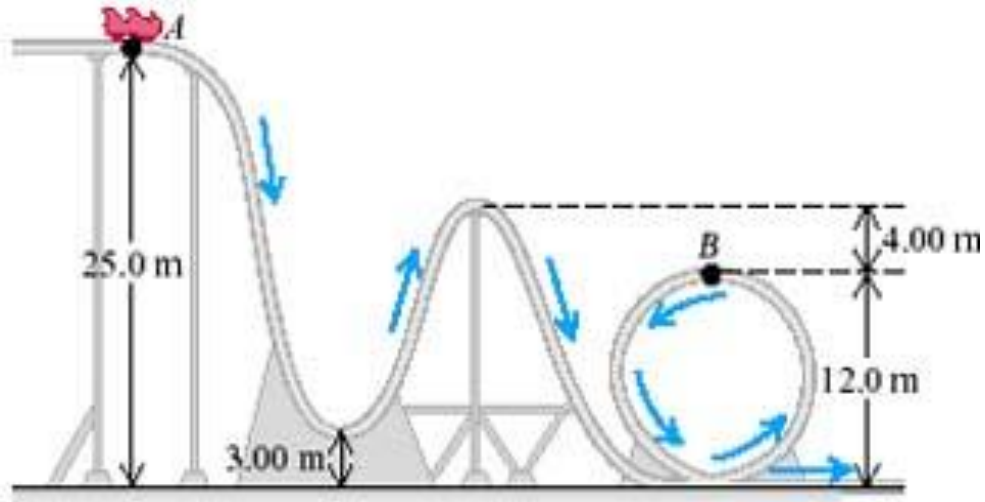
- A decent spy-satellite camera can distinguish that subtends angles of little as $0.5 \times 10^{-6} \text{ rad}$. How small an object could such a device detect from 300 km away?
- It takes 30.0 s for the shaft of a big electric motor to reach 500 rev/sec. What is its average angular acceleration ?

PRACTISE QUESTIONS CONTINUE...

- In 2002, professional skateboarder Bob Burnquist became the first to successfully navigate a 360° full pipe turn. Determine the minimum speed which would be required at the top of the circular loop to make it through the 1.8-m radius pipe.
- A loop de loop track is built for a 938-kg car. It is a completely circular loop - 14.2 m tall at its highest point. The driver successfully completes the loop with an entry speed (at the bottom) of 22.1 m/s.
 - a.** Using energy conservation, determine the speed of the car at the top of the loop.
 - b.** Determine the acceleration of the car at the top of the loop.
 - c.** Determine the normal force acting upon the car at the top of the loop.

PRACTISE QUESTIONS CONTINUE...

- Give speed at each critical points and final speed which the car(350 kg) will have after a sequence of loops as shown..



- The rollercoaster moves freely without negligible friction. The radius of the loop is 20m and the car barely makes it through the loop.

- Find the speed at position 3.
- Find the speed at position 1 and 2.
- Find the difference in height between position 1 and 4 if the speed at position 4 is 10m/s.

