E4 - Fission

Oxford solution

Worked example 1

In neutron-induced fission, a nucleus of uranium-235 ($^{235}_{92}U$) absorbs a neutron and yields a nucleus of xenon-140 ($^{140}_{54}Xe$) and a nucleus of strontium-94 ($^{94}_{38}Sr$), according to the reaction

$$^{235}_{92}U + ^{1}_{0}n \rightarrow ^{140}_{54}Xe + ^{94}_{38}Sr + x\,^{1}_{0}n$$

a. Calculate the number of neutrons released in the reaction.

The atomic masses of the nuclides are given in the table.

b. Calculate, in MeV, the energy released in the reaction.

c. Estimate, in J, the nuclear energy transferred when 1.0 kg of pure uranium-235 undergoes fission.

Nuclide	Atomic mass
$^{235}_{92}U$	235.0439 u
$^{140}_{54}Xe$	139.9216 u
$^{94}_{38}Sr$	93.9154 u

Worked example 2

Xe-140 and Sr-94 produced in neutron-induced fission of uranium-235 in Worked example 1 are radioactive and undergo further decays. The stable end products of their respective chains of decays are cerium-140 ($^{140}_{58}Ce$) and zirconium-94 ($^{94}_{40}Zr$).

a. Explain why the combined proton number of Ce-140 and Zr-40 is different from the proton number of U-235.

The binding energies per nucleon are given in the table.

b. Calculate, in MeV, the total energy released as a result of this fission reaction of U-235.

Nuclide	Binding energy / *A*
$^{235}_{92}U$	7.591 MeV
$^{140}_{58}Ce$	8.376 MeV
$^{94}_{40}Zr$	8.667 MeV

1. Consider the neutron-induced fission reaction

$$^{235}_{92}U + ^{1}_{0}n \rightarrow ^{144}_{56}Ba + ^{89}_{36}Kr + 3\,^{1}_{0}n$$

The following data are given about the binding energies per nucleon of these nuclides.

Nuclide	Binding energy / *A*
$^{235}_{92}U$	7.591 MeV
$^{144}_{56}Ba$	8.265 MeV
$^{89}_{36}Kr$	8.615 MeV

a. Calculate, in J, the energy released in the reaction.

b. Estimate the fraction of the mass of uranium-235 converted to energy in this reaction.

c. A nuclear power station outputs 1.3 GW of electrical power. Use your answer in part b. to estimate the mass of uranium-235 that undergoes fission in one day. Assume that the overall efficiency of nuclear to electrical energy transfer in this power station is 0.30.

2. Consider neutron-induced fission of plutonium-239,

$$^{239}_{94}Pu + ^{1}_{0}n \rightarrow ^{134}_{54}Xe + ^{103}_{40}Zr + x\,^{1}_{0}n$$

a. State the number of neutrons released in this reaction.

b. Calculate, in MeV, the energy released. The following data are available for this question.

Nuclide	Atomic mass
$^{239}_{94}Pu$	239.0522 u
$^{134}_{54}Xe$	133.9054 u
$^{103}_{40}Zr$	102.9272 u

W/1 - pg

a) $235 + 1 = 140 + 94 + x \Rightarrow x = 2$

b)

$$\left\{ {}^{235}_{92}U + {}^{1}_{0}n \rightarrow {}^{140}_{54}Xe + {}^{94}_{38}Sr + 2\,{}^{1}_{0}n \right\}$$

$$\Big\{ m_U + m_n \rightarrow m_{Xe} + m_{Sr} + 2m_n$$

$$\Rightarrow \Delta m = m_U - m_{Xe} - m_{Sr} - m_n \Big\}.$$

$\Delta m = 235 - 139.9216 - 93.915 - 1.0087$

$= 0.1982\ u$

$E = \Delta m \times 931.5$ MeV

$= 0.1982 \times 931.5 = 184.6$ MeV.

c) fraction of Uranium converted into energy $= \dfrac{0.1982}{235.0439} = 8.4 \times 10^{-4}$

for 1 kg $\rightarrow 1 \times 8.4 \times 10^{-4}$ kg will be converted to energy

$\Rightarrow$ Energy transfered $= mc^2$

$= 8.4 \times 10^{-4} \times (3 \times 10^8)^2$

$= 7.6 \times 10^{13}$ J.

w/2 → pg 671

a) There could have been beta decay.

b) $\Delta E = 140(8.376) + 94(8.667) - 235(7.591)$

$= 203.5$ MeV

p/1 → pg 671

$^{235}_{92}U + ^{1}_{0}n \rightarrow ^{144}_{56}Ba + ^{89}_{36}Kr + 3\,^{1}_{0}n$

a) $\Delta E = 144(8.265) + 89(8.615) - 235(7.591)$

$= 173$ MeV $= 17\cancel{6} \times 10^{6} \times 1.6 \times 10^{-19}$ J

$= 2.72 \times 10^{-11}$ J

b) corresponding mass defect

$\Delta m = \frac{173}{931.5} = 1.857 \times 10^{-1}$ u

fraction of uranium $= \frac{1.857\ 10^{-1}}{235.049}$

$= 7.9 \times 10^{-4}$

c) $P = \frac{E}{t}$, $E = P \times t$

$= 1.3 \times 10^9 \times 60 \times 60 \times 24$

$= 1.1 \times 10^{14}$ J / day

efficiency = 0.30

$\Rightarrow$ energy input $= \frac{1.1 \times 10^{14}}{0.30}$

$= 3.66 \times 10^{14}$ J.

corresponding mass $= \delta m = \frac{3.66 \times 10^{14}}{(3 \times 10^8)^2} = 4.067 \times 10^{-3}$ kg

1 kg 7.9×10^{-4} converted to energy

$\Rightarrow$ mass of uranium needed $\Big\} = \frac{4.067 \times 10^{-3}}{7.9 \times 10^{-4}}$

$= 5.147$

$= 5.1$ kg

P/2 - Pg 671

$^{239}_{94}U + ^{1}_{0}n \rightarrow ^{134}_{54}Xe + ^{103}_{40}Zr + x\,^{1}_{0}n$

a) $239 + 1 = 134 + 103 + x$

$\Rightarrow$ $x = 3.$

b) $\Delta m = 239.0522 - 133.9054 - 102.9272 - 2(1.0087)$

$\Delta m = 0.2022\ u$

$E = \Delta m \times 931.5 = 0.2022 \times 931.5$

$= 188\ MeV$

Data based Questions.

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Data-based questions

The number of neutrons produced from the fission of U-235 varies. The table below gives the probability *P* of different numbers of neutrons *N* being released.

N	*P*
0	0.033
1	0.174
2	0.335
3	0.303
4	0.123
5	0.028
6	0.003

- Calculate the average number of neutrons emitted (express your answer to 1 decimal place).
- Theory suggests that $\ln P = k(N - N_{av})^2 + A$ where N_{av} is the average number of neutrons emitted and *k* and *A* are constants. Plot a graph of ln *P* against $(N - N_{av})^2$ to test this theory.
- Use your graph to find values of *k* and *A*. Hence, find the probability of a fission reaction releasing 7 neutrons.

Data from N. Ensslin: *The Origin of Neutron Radiation*

N	P	lnp	$(N - N_a)^2$
0	0.033	-3.411	1.21×10^{-2}
1	0.174	-1.748	7.92×10^{-1}
2	0.335	-1.093	3.57×10^{0}
3	0.303	-1.194	8.35×10^{0}
4	0.123	-2.095	1.51×10^{1}
5	0.028	-3.575	2.39×10^{1}
6	0.003	-5.809	3.46×10^{1}
Av	0.11		

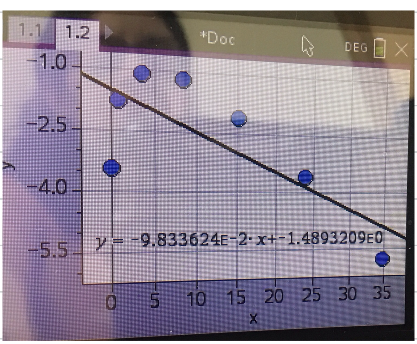
for $N = 7$

$(N - N_a)^2 = 4.74 \times 10^1$

$\Rightarrow \ln P = -6.141$

$\Rightarrow P = 0.00215$

$\therefore$ for $N = 7$, $P = 0.00215$



Eq of Regression

$\ln P = k(N - N_a)^2 + A$

Compare with

$y = mx + c$

$\ln P = -0.098(N - N_a)^2 - 1.489$