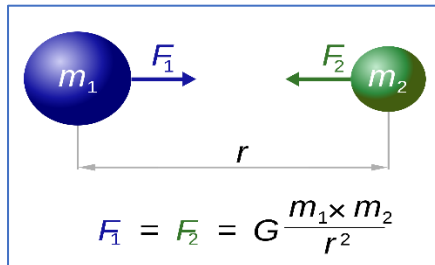


NEWTON'S UNIVERSAL LAW OF GRAVITATION :

Every single point mass attracts every other point mass with a force that is directly proportional to the product of their masses and inversely proportional to the square of separation. If two point masses m_1 and m_2 are separated by a distance r , then force of attraction F between them is given by : $F \propto \frac{m_1 m_2}{r^2} =$

$$G \frac{m_1 m_2}{r^2}, G \text{ is universal gravitational constant} = 6.6742 \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}$$



GRAVITATIONAL FIELD : It is a region of space where a mass experience a force because of its mass.

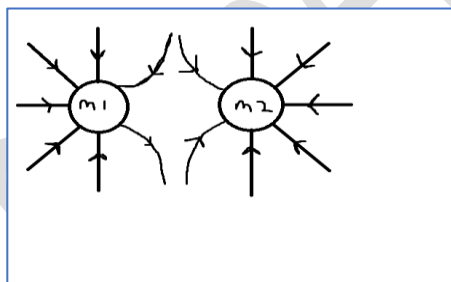
GRAVITATIONAL FIELD STRENGTH : It gives a measure of how much force a body will experience in the gravitational field.

It is defined as : The force per unit mass experienced by a small point mass placed in the field.

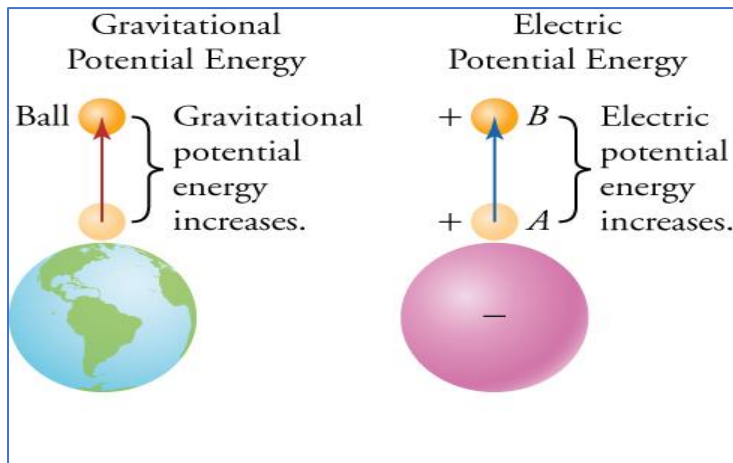
$$g = \frac{F}{m} = G \frac{m}{r^2} \text{ N kg}^{-1}, m \text{ is the test mass, } g \text{ is a vector quantity}$$

Field strength caused by several bodies is added vectorially.

NOTE : The reason for using a small test mass is because a big mass might change the field that you are trying to measure.



FIELD LINES : They are drawn in the direction that the mass would accelerate if placed in the field. The arrows give the direction of the field. The field strength is given by the density of the lines.

**GRAVITATIONAL POTENTIAL ENERGY**

: As we move away from the surface of the earth PE increases. $PE = mgh$, here gh answers the question 'where' and is called the gravitational potential V

GRAVITATIONAL POTENTIAL : It is the work done per unit mass in taking a small point mass from zero potential (surface of the earth) to the point in question.

Suppose you are to bring a mass m from infinity to point as shown ;

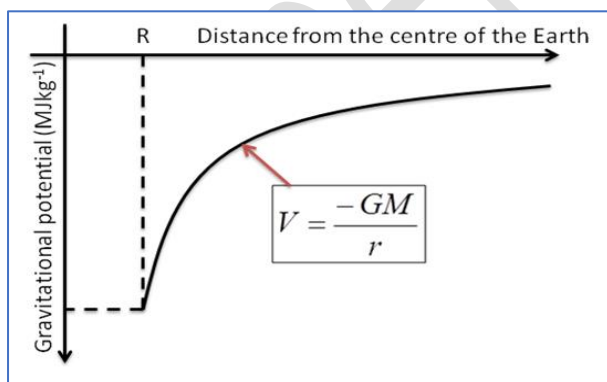
$$F = G \frac{Mm}{r^2} \text{ and}$$

$$\text{Work done} = F \times r = G \frac{Mm}{r}$$

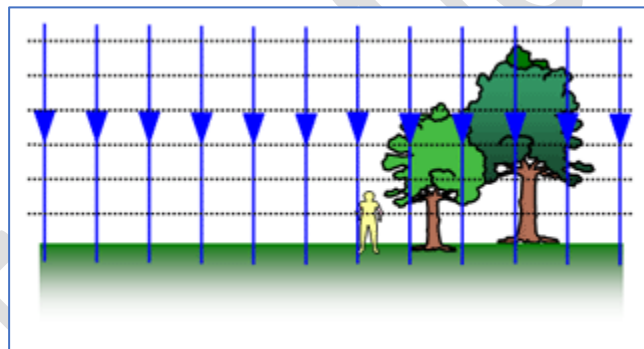
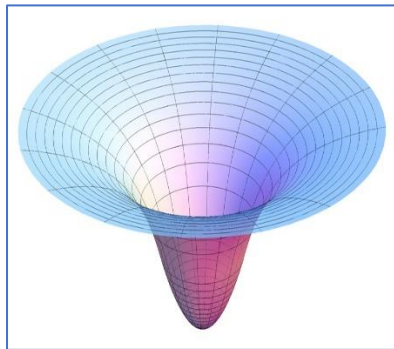
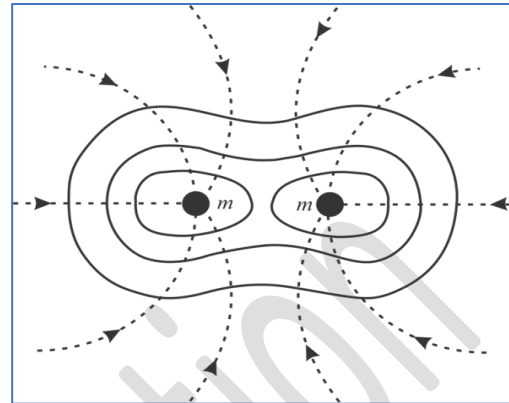
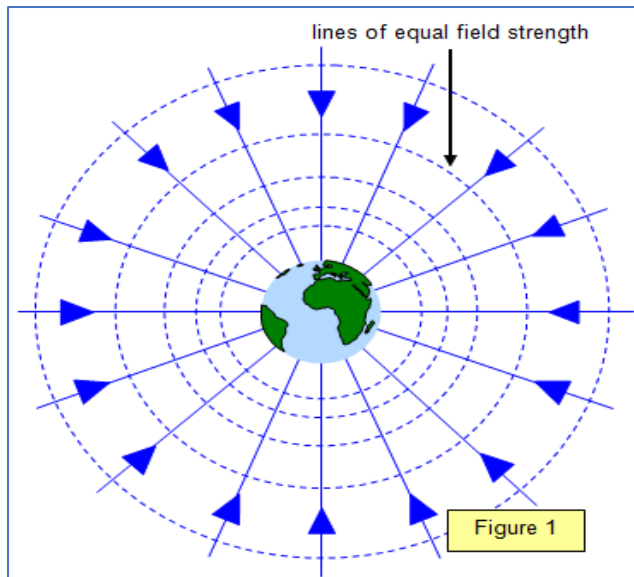
$$V = \text{Work done per unit mass}$$

$$= -G \frac{M}{r},$$

negative sign because the mass is pulled to the earth on its own due to the force of gravity. Or the direction of force applied by you is opposite to the direction of motion from infinity to that point. Potential is a scalar quantity, so adding potential is a matter of adding magnitude.



EQUIPOTENTIAL LINES : By joining all positions of the same potential ($V = gh = \text{same}$) we get a line of equal potential and these lines are called equipotential lines. Gravitational field lines are always perpendicular to the equipotential lines. These lines are closest together where the field is strongest. In 3D they would form equipotential surfaces.



POTENTIAL GRADIENT :

As we move above the surface of the earth, the amount of work done is change in PE.

$$\text{Hence } WD = \Delta PE = mg\Delta h$$

again $V = \text{work done per unit mass}$

$$\Rightarrow \text{work done} = V \times m$$

therefore equating both the equations,

$$m\Delta V = mg\Delta h$$

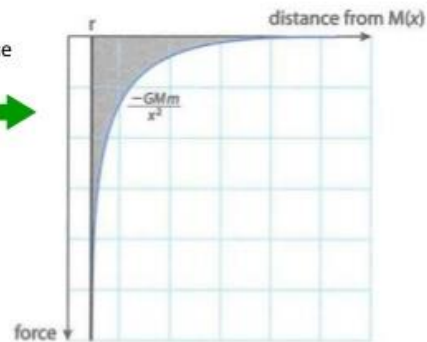
$$\Rightarrow g = \frac{\Delta V}{\Delta h}$$

$\Rightarrow \text{potential gradient} = \text{the field strength}$

Gravitational Potential (V) Equation

$$F = G \frac{Mm}{r^2}$$

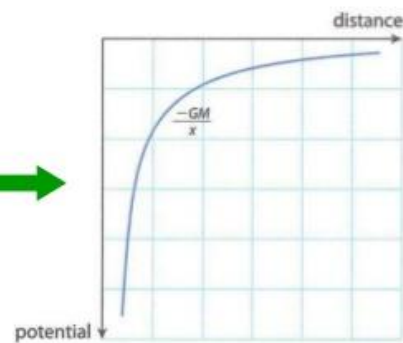
Negative because the force is to move from ∞ to r



The area under a force-distance graph is the same as the Work Done so it can be used to calculate Potential.

The equation is:

$$V = -G \frac{M}{r}$$



ESCAPE SPEED : If we throw the ball up really fast, it might get so high that the gravitational field strength would start to decrease.

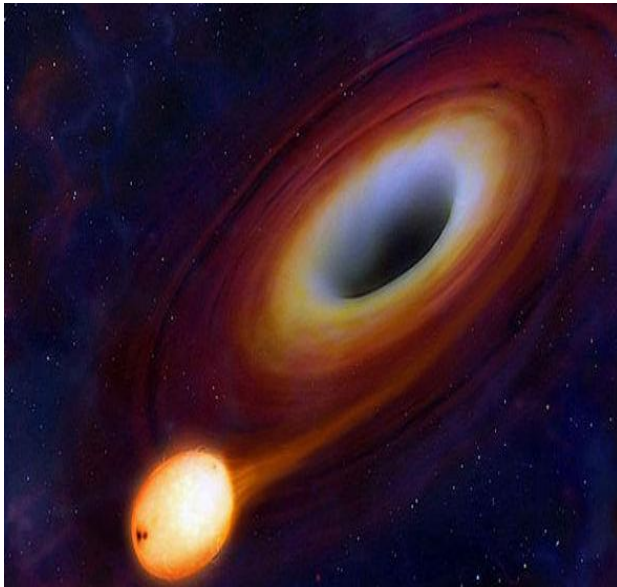
So loss in kinetic energy = gain in PE

$$\Rightarrow \frac{1}{2}mv^2 = -G \frac{Mm}{r_2} - \left(-G \frac{Mm}{r_E} \right), \text{ here } r_2 \text{ is infinity}$$

$$\Rightarrow \frac{1}{2}mv^2 = -G \frac{Mm}{\infty} - \left(-G \frac{Mm}{r_E} \right)$$

$$\Rightarrow \frac{1}{2}mv^2 = \left(G \frac{Mm}{r_1} \right),$$

$$\Rightarrow v_{\text{escape}} = \sqrt{\frac{2GMm}{r_E}}$$

BLACK HOLES :

A black hole is a place in space where gravity pulls so much that even light can not get out. The gravity is so strong because matter has been squeezed into a tiny space. The reason this force doesn't cause the star to collapse is that the particles are given KE due to nuclear reactions taking place. As time passes the nucleus gas gets used up and starts start to collapse. As this happens the escape velocity is so big that even light cannot escape and the star has formed a black hole. This can happen when a star is dying.

Because no light can get out, people can't see black holes. They are invisible. Space telescopes with special tools can help find black holes.

KEPLER'S LAWS : Kepler's three laws of planetary motion can be stated as follows:

Kepler found a relationship between the time it took a planet to go completely around the sun (T , sidereal year), and the average distance from the sun (R , semi-major axis)...

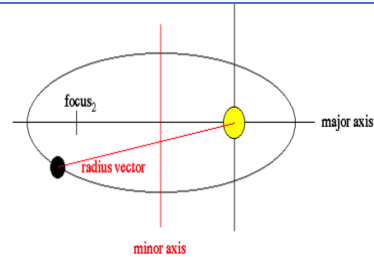
$$\frac{T_1^2}{T_2^2} = \frac{R_1^3}{R_2^3} \quad \left(\begin{array}{l} T^2 = T \times T \\ R^3 = R \times R \times R \end{array} \right)$$

THIRD LAW OF PLANETARY MOTION

- (1) All planets move about the Sun in elliptical orbits, having the Sun as one of the foci.
- (2) A radius vector joining any planet to the Sun sweeps out equal areas in equal lengths of time.
- (3) The squares of the sidereal periods (of revolution) of the planets are directly proportional to the cubes of their mean distances from the Sun.

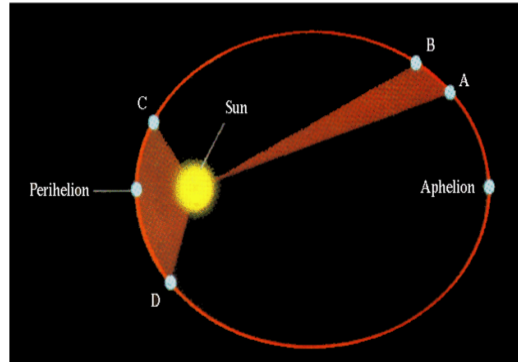
First Law

Each planet moves in an elliptical orbit with its star (Sun) at one focus



Second Law

(law of equal areas): an orbiting object will take the same amount of time to travel between points A & B as it takes to travel between points C & D



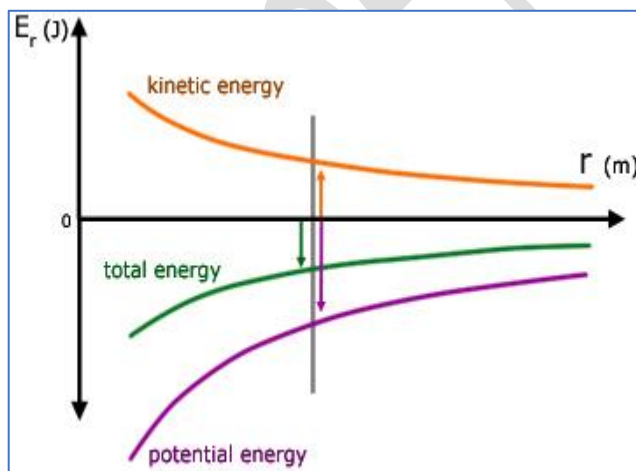
Third Law

(law of harmonics): The square of a planet's orbital time is proportional to its average distance from the star (Sun) cubed.

ENERGY OF AN ORBITING BODY :

$$PE = -G \frac{Mm}{r}$$

and $KE = \frac{1}{2}mv^2$. if we approximate the orbits to be circular,



then equating centripetal force and gravity gives:

$$G \frac{Mm}{r^2} = \frac{mv^2}{r}$$

rearranging and multiplying by $\frac{1}{2}$ gives :

$$\frac{1}{2}mv^2 = G \frac{Mm}{2r}$$

$$\Rightarrow KE = G \frac{Mm}{2r}$$

now Total energy = PE + KE

$$\begin{aligned}\Rightarrow TE &= -G \frac{Mm}{r} + G \frac{Mm}{2r} \\ &= G \frac{Mm}{r}\end{aligned}$$

- Low satellites have greater kinetic energy but less total energy than distant satellites. So although the distant ones move with slower speed, work has to be done to increase the orbital radius. Moving the other way, to move from distant orbit to a close orbit, the spaceship needs to lose energy.
- The satellites closer to the earth have a time period much shorter (velocity more) than the distant ones. So a low spy satellite could orbit the earth faster than a much higher TV satellite.